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3 Dimensional CFD analysis of Laminar flow Natural Convection of Hollow Cylinder with Annular Fins

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Abstract - Natural Convection heat transfer from a horizontal hollow cylinder having annular fins on the outer surface has been studied numerically. The constant temperature boundary condition was applied at the inner surface of the hollow cylinder. Boussinesq approximation was used with steady and laminar flow. For simulation ANSYS FLUENT was used to solve the discretized system and three dimensional model was prepared in SOLIDWORKS. The present investigation showed us that the analytical solution and numerical solution for unfinned hollow cylinder are very accurate and the cases with fins from 4 to 20 were solved by simulated showing us the pictorial representation of temperature plume and velocity vectors. The plots are shown comparing different thermal parameters such has heat flux, total heat transfer, Rayleigh number, Nusselt number and effectiveness of the system with fins with respect to the number of fins attached to the hollow cylinder.

Keywords: Fins, Natural, Convection, Heat, Transfer.

1. Introduction

Convection heat transfer is complicated in itself since, it incorporates two principles i.e. heat conduction and fluid motion. Fluid motion is the key enhancement factor; higher the fluids velocity, higher the heat transfer rate. There are two type of convections : Natural Convection and Forced convection. Forced convection is one where the fluid is made to flow via some external forces and Natural convection is the one which is governed by Buoyancy forces. Natural convection plays a very important role in almost every other industry like cooling of equipment such as microprocessors, TVs, transistors and microchips, heat dissipation from cooling coils and power transmission lines etc.

In cases of natural convection in thermal systems, uses of fins is a smart method to enhance the heat transfer rate significantly, as it was studies by Welling et al. [1], Harahap et al. [2], Singh and Dash [3] and Kraus et al. [4]. The natural convection has no moving components hence, it is more feasible to use where the structural integrity of the equipment to be cooled is under consideration. There are not many significant numerical studies performed by many of the researchers all across the globe.

Senapati et al. [5] did numerical study on vertical cylinder with annular fins where they studied conjugated heat transfer characteristics for optimization of best fin spacing for the case of turbulent flow natural convection and they also derived a correlation between Nusselt number and geometrical parameters along with Rayleigh number. Senapati et al. [6] (2016) did three dimensional numerical study on the effects of shifting the annular fins on horizontal cylinder along vertical axis in certain ratios, they concluded with giving the temperature plumes and flow characteristics for laminar flow natural convection. They also derived a correlation between Nusselt number and geometrical parameters and also the Rayleigh number using regression analysis. One of the significant numerical studies was conducted by Nada et al. [7] where they analysed the effects of geometry, arrangement, dimension and number of annular and longitudinal fins on horizontal tube arrangement in an enclosure. Result showed that the heat flux increased as the Rayleigh number increase and also with increasing in annulus area, number of fins and fin width; for same area of fins longitudinal fins were more effective than annular fins. The convection heat transfer on horizontal hollow cylinder with internal axial fins was investigated by Acharya et al. [8]. They performed three dimensional numerical simulation for a short cylinder ($L/D<1$) where they found that the heat rejection from the surface of the cylinder can be optimized (maximized) for some level height and certain fin number after that it will reduce, for long cylinder($L/D>1$) [8] the maximum value of heat transfer will vanish for all given length of the fins and spacing between fins. Sun et al. [9] analysed very intensively the natural convection along the vertical cylinder heat sinks with longitudinal fins where they discussed the thermal boundary layer along with the flow patterns of the air,

Nomenclature

$\vec{V}$: Velocity of fluid in vector form, $\frac{m}{s}$ α : Thermal diffusivity, $\frac{m^2}{s}$ ρ : Density, $\frac{kg}{m^3}$ $\vec{g}$: Acceleration due to gravity, $\frac{m}{s^2}$ p : Free stream pressure, $\frac{N}{m^2}, Pa$ T_f : Film temperature , ℃, K T_w : Wall temperature, ℃, K Ra_i : Rayleigh number based on fin spacing, $Ra_s = \frac{g\beta(T_w - T_f)S^3}{\vartheta\alpha}\left(\frac{S}{D_o}\right)$ T_∞ : Free stream temperature, ℃, K Q : Total heat transfer, W h : Convective heat transfer coefficient, $\frac{W}{m^2-K}$ A_t : Total outer surface area of the tube, m^2 Pr : Prandtl Number, $Pr = \frac{\mu C_p}{K}$ D_f : Diameter of the annular fins, m L_f : Length of the fins, $L_f = \frac{D_f - D_o}{2}, m$ q_{cfd} : Heat Flux based on CFD result, $\frac{W}{m^2}$ Nu_{cfd} : Nusselt number based on CFD result $\in$: Effectiveness of the system	T : Temperature of the fluid, ℃, K D_o : Outer diameter of the tube, m D_i : Inner diameter of the tube, m R : Specific gas constant, $\frac{J}{kgK}$ T_s : Surface temperature, ℃, K p_{atm} : Atmospheric pressure, $\frac{N}{m^2}, Pa$ Ra : Rayleigh number, $Ra = \frac{g\beta(T_w - T_f){L_c}^3}{\vartheta\alpha}$ i : Suffix of Rayleigh number signifying the number of fins S : Fin spacing, m Nu : Nusselt number, $Nu = \frac{hD_o}{k}$ μ : Dynamic Viscosity, $\frac{kg}{m-s}$ ϑ : Kinematic Viscosity, $\frac{m^2}{s}$ N : Number of fins C_p : Specific heat capacity at constant pressure, $\frac{KJ}{kgK}$ K : Thermal conductivity , $\frac{W}{m-K}$ h_{cfd} : Convective heat transfer coefficient based on CFD result, $\frac{W}{m^2-K}$ q_{inner} : Heat flux at the inner wall based on CFD result, $\frac{W}{m^2}$ β : Coefficient of thermal expansion, $\frac{1}{K}$

they also investigated the values of local Nusselt number distribution patterns along the fin length and cylinder surface[9]. They discovered that the average Nusselt number along the length of the fin is approximately proportional to the exponential function of the vertical distance with deviation of ±10%.

2. Analytical and Mathematical Modelling

A horizontal cylinder of outer diameter (D_o) 25mm and inner diameter (D_i) 20 mm with length of the fins (L_f) 10 mm is taken into analysis firstly without fins and then with a number of fins N the schematic side view of the system is shown in Fig. 1 where as the Fig. 2 shows us the three dimensional model.

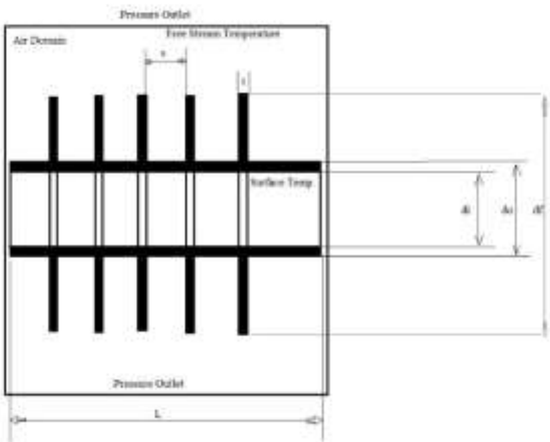


Fig. 1: Schematic Representation.

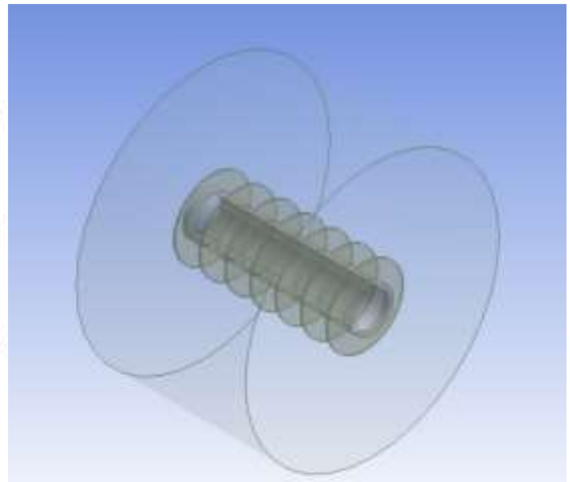


Fig. 2: Three Dimension Model Representation.

In the current study we have done the computational analysis for 10 cases with N= 0,4,6,8,10,12,14,16,18,20 along with complete analytical validation of the cases. We have selected the material of the tube i.e. Aluminium for its high thermal conductivity. The thermal parameters have been reported in their standard form. The model is in three dimensional isometric view as it is very clear from Fig. 2, the computational investigation is also done in three dimensional form.

A circular Outside air domain was chosen with a constant free stream temperature and the diameter of the domain is 150 mm. Boundary conditions employed were as follows, the temperature of inner wall of the hollow cylinder was kept constant $T_s = 398\ K$ which; is normally constant in the case of industrial application; and is fixed for all cases, As done by Senapati [6]. Since, aluminium has very high thermal conductivity the temperature drop along the radial direction of tube was ignored [8]; also with the conductive thermal resistance of the hollow cylinder.

The governing equations in differential for the analytical solution we have taken the flow to be steady, laminar and incompressible ideal gas so the assumptions are written as follows:

Energy equation :

$$(\vec{V}.\nabla)T = \alpha(\nabla^2 T) \tag{1}$$

Momentum equation :

$$\rho(\vec{V}.\nabla)\vec{V} = \rho\vec{g} - \nabla p + \mu(\nabla^2\vec{V}) \tag{2}$$

Continuity equation :

$$\nabla.\vec{V} = 0 \tag{3}$$

Equation of state :

$$p = \rho RT \tag{4}$$

Boundary conditions :
Constant temperature condition : at the inner wall of hollow cylinder

$$T = T_s \tag{5}$$

At the hollow cylinder outer wall : No Slip Condition

$$\vec{V}.\hat{n} = 0 \tag{6}$$

The cylinder wall is considered to be at a constant temperature (Fig. 1) at the inner surface since, the problem was formulated whilst taking into consideration of phase change material which takes place at constant temperature condition. Since, the cylinder has some thickness it will experience some drop in temperature as it reaches the outer surface however, the thermal conductivity is high for aluminium the temperature at outer wall was calculated to come $T_{ow} = 397.97\ K$ which is almost equal to the inner wall temperature.

At the pressure outlet, atmospheric pressure is considered and if there is any generated backflow it will have the free stream temperature.

$$p = p_{atm}\ ; p_{gauge} = 0\ MPa\ ; T = T_{\infty} \tag{7}$$

Energy equation after applying the necessary condition that will be used for conduction in the fins :

$$\nabla^2 T = 0 \tag{8}$$

Equation (8) will be solving the heat conduction for the fin. This equation requires two boundary condition one of which is the temperature at the base and other no heat conduction at the tips. However, the solution of equation for annular fins is more typical then the rectangular.

For the case of simple hollow cylinder with keeping the base temperature constant the heat transfer is calculated by empirical correlations derived by Churchill and Chu [10], equation (14). The surrounding fluid is air, the fluid properties will be taken at the film temperature T_f,

$$T_f = \frac{T_w + T_{\infty}}{2} \tag{9}$$

Rayleigh number (which is on the basis of the tube outer diameter) is calculated on the basis of the following relation:

$$Ra = \frac{g\beta(T_w - T_f){D_o}^3}{\vartheta\alpha} \tag{10}$$

The Rayleigh number on the fin spacing for the finned cylinder with fin spacing (S) is calculated by the relation :

$$Ra_s = \frac{g\beta(T_w - T_f)S^3}{\vartheta\alpha}\left(\frac{S}{D_o}\right) \tag{11}$$

From the newton's law of cooling, the total heat transfer from the outer surface is :

$$Q = hA_t(T_w - T_{\infty}) \tag{12}$$

Where, the average heat transfer coefficient (h) is calculated from the definition of Nusselt number :

$$Nu = \frac{hD_o}{k} \tag{13}$$

for the fluid and the Nusselt number is being calculated from the Churchill's relation :

$$Nu = \left[0.6 + \frac{0.387(Ra)^{1/6}}{\left\{1+\left(\frac{0.559}{Pr}\right)^{9/16}\right\}^{8/27}}\right]^2 \tag{14}$$

On solving for the case of simple tube and with the free stream temperature $T_\infty = 298K$, the Rayleigh number is $Ra = 70363.21$, which is well within the range of laminar flow natural convection ($10^4 \leq Ra \leq 10^7$). The Nusselt number using equation (14) gives the value $Nu = 7.81$. Now with use of equation (13) average heat transfer coefficient h is calculated coming out to be $h = 9.38\ \frac{W}{m^2K}$. Finally, the heat flux is calculated to be $\dot{q} = 938.11\ \frac{W}{m^2}$.

The governing equations from (1)-(5) is integrated over the control volume and then discretized by finite volume method. The computational software used is Ansys FLUENT by imposing the boundary conditions (6)&(7). The SIMPLE scheme has been used with pressure body weighted forced and second order upwind for energy. The final values obtained after the simulation for 1707k (17 lakhs) cells the heat flux was $\dot{q}_{cfd} = 936.79\ \frac{W}{m^2}$, with heat transfer coefficient $h_{cfd} = 9.36\frac{W}{m^2K}$ and the Nusselt number $Nu_{cfd} = 7.80$. Therefore, the accuracy between the simulation and analytical calculation came out to be very high with error of only 0.14%. The grid independence test was also performed as shown in Fig. 3. The results showed us that the heat flux values coming from the solution performed by Ansys FLUENT are dropping initially and then they became steady after the 17 lakhs elements and the heat flux at the inner wall is $\dot{q}_{inner} = 1171.03\ \frac{W}{m^2}$

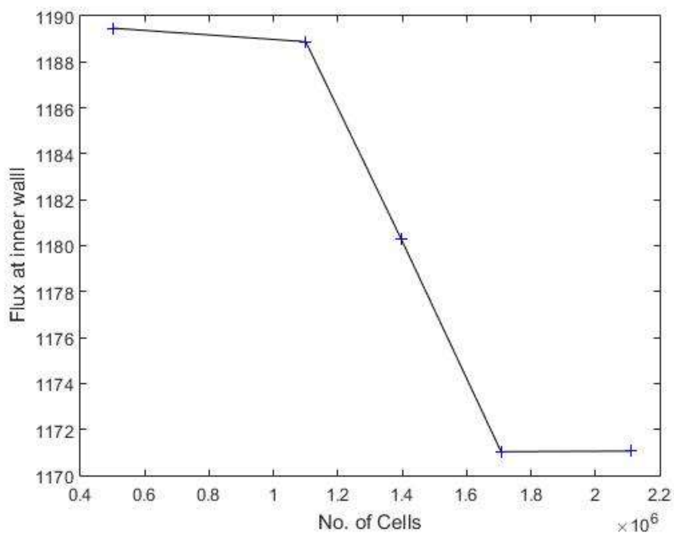


Fig. 3: Grid Independence Test.

3. Result and Discussion

After verifying the simulation for unfinned hollow cylinder we went forward with the CFD simulation for the cases of N=0,4,6,8,10,12,14,16,18,20. The temperature plume and the velocity profile for the standard unfinned cylinder case are shown in fig. 4 and fig. 5 respectively.

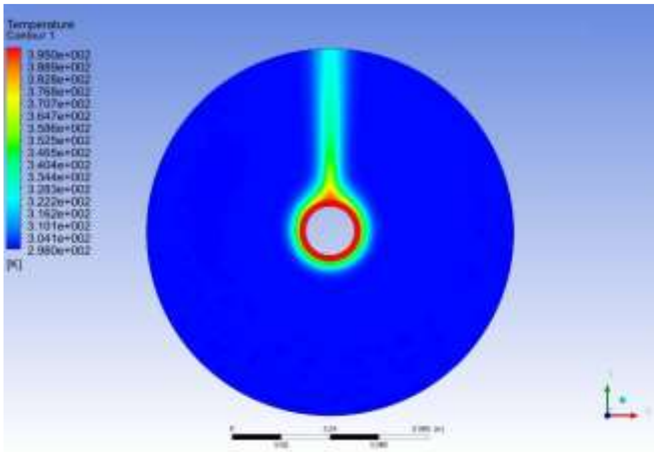


Fig. 4: Temperature Contour for Cross section.

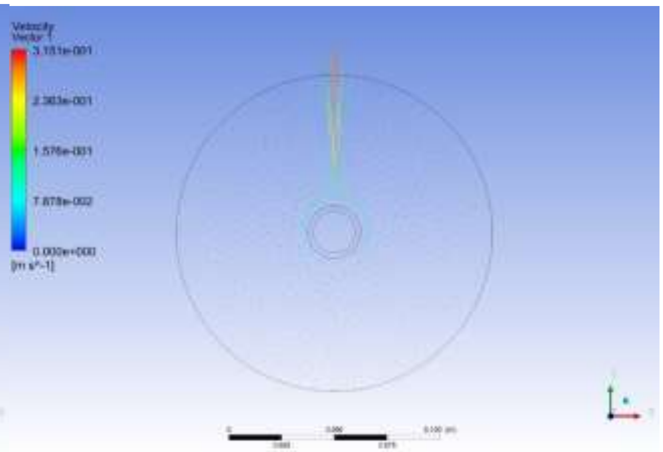
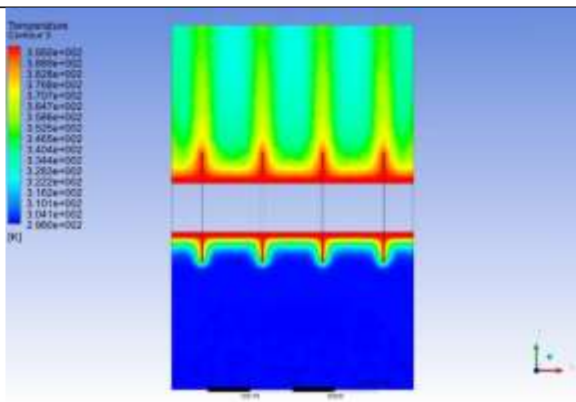
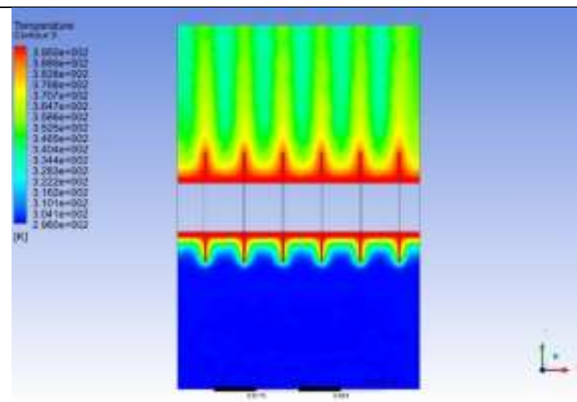
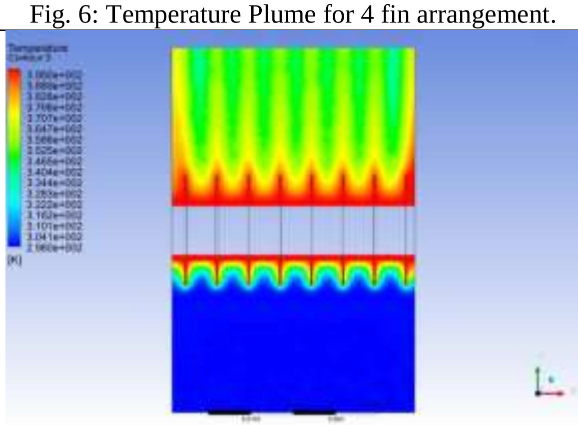
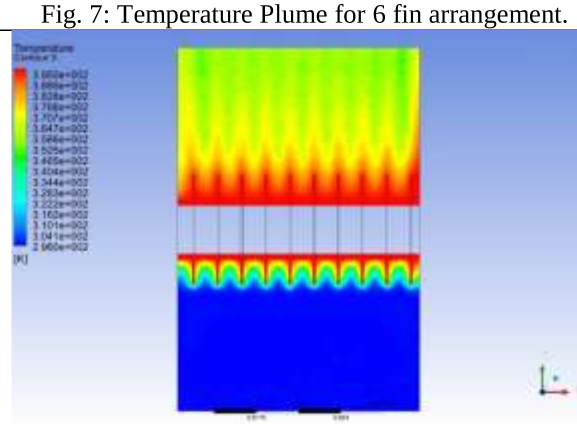


Fig. 5: Velocity Profile for cross section.

The temperature plumes for the rest of the cases are shown in table 2 containing fig. 6-14.

Table 2: Showing temperature plumes along the length for different cases.

 Fig. 6: Temperature Plume for 4 fin arrangement.	 Fig. 7: Temperature Plume for 6 fin arrangement.
 Fig. 8: Temperature Plume for 8 fin arrangement.	 Fig. 9: Temperature Plume for 10 fin arrangement.

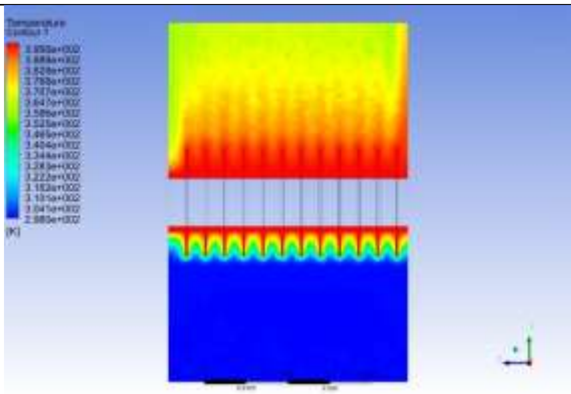


Fig. 10: Temperature Plume for 12 fin arrangement.

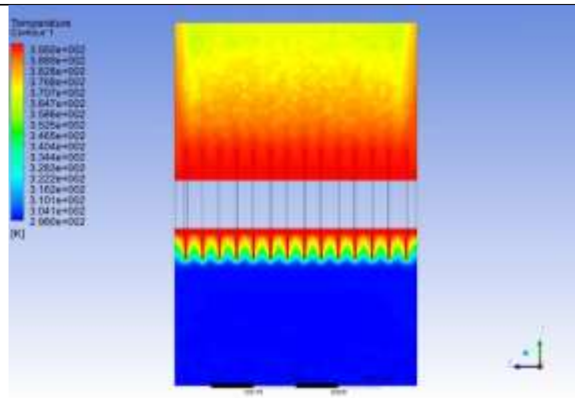


Fig. 11: Temperature Plume for 14 fin arrangement.

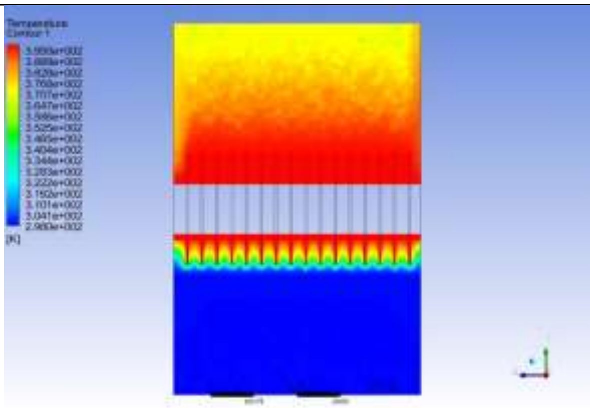


Fig. 12: Temperature Plume for 16 fin arrangement.

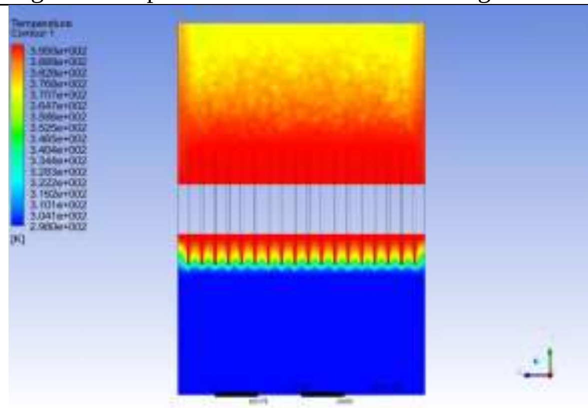


Fig. 13: Temperature Plume for 18 fin arrangement.

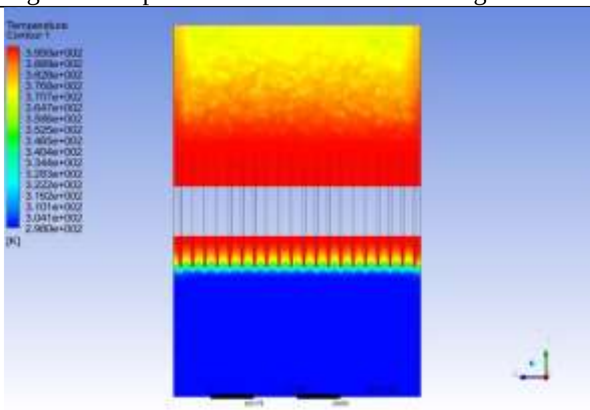


Fig. 14: Temperature Plume for 20 fin arrangement.

The Fig. 15 shows the relation between Rayleigh number and the number of fins.

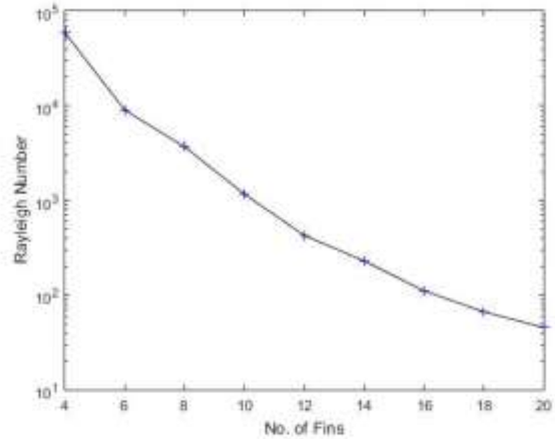


Fig. 15: Rayleigh No. vs No. of fins.

It can be deducted from the plot that for the same boundary conditions the Rayleigh number is initially dropping steadily therefore the y scale had to be taken in logarithmic scale. The Rayleigh number on the basis of fin spacing for 10 fins $Ra_{10} = 1922.97$ values going further down till Rayleigh for 20 fins $Ra_{20} = 45.479$. The following Fig. 16 shows the plot of Nusselt number and the number of fins.

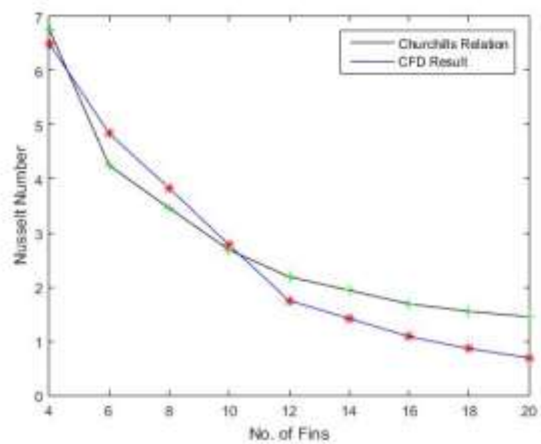


Fig. 16: Nusselt no. vs No. of fins.

There are two plots, one of the values of the Nusselt number calculated from the (Churchill's relation) equation (14) and the other plot is from the CFD data. Both the curves area showing gradual drop in the Nusselt number values. The fig. 17 and fig. 18 shows the plot for heat flux and the total heat transfer (from the cylinder to the outside air). From the plot of heat flux fig. 17 it is clear that the flux is fluctuating as the no. of fins is increasing till the number of fins are 12, then the heat flux decreased with the increase in number of fins hence, for the designing of such systems the number of fins can be selected on the basis of one's requirement of the heat flux. The fig. 18 shows us that even though heat flux is decreasing but the total heat transfer is continuously increasing, it is because the total surface area of the system is increasing even though the heat transfer coefficient is gradually decreasing as it can be deducted from the heat flux plot and equation (12).

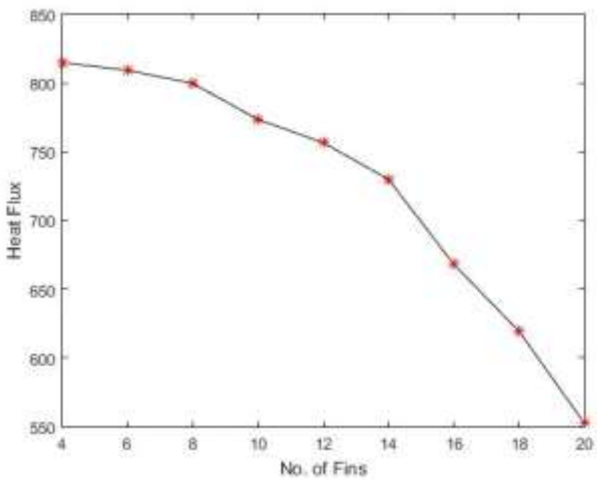


Fig. 17: Heat Flux vs No. of Fins.

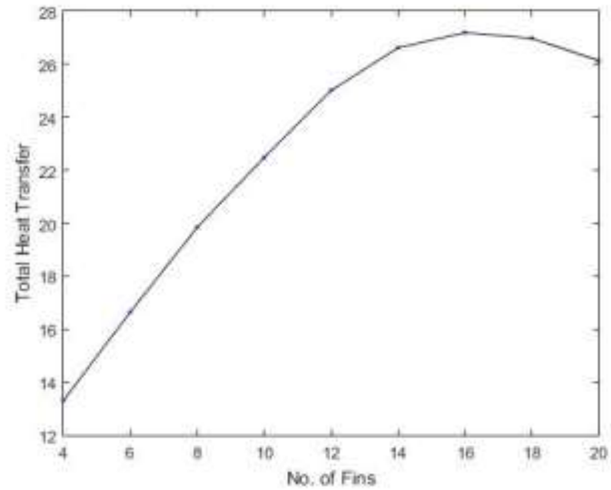


Fig. 18: Total Heat transfer vs No. of fins.

It can be inferred that the scope of optimizing is limited since, the total heat transfer is increasing with the increasing in number of fins till the 16 fins and then the total heat transfer started to decrease as we increasing the fins from 16 to 18 and 20. Moreover, the heat flux is dropping as the number of fins are increasing. The next fig. 19 is showing the most significant result of the research, the plot of effectiveness with respect to the number of fins. It can also be deduced that, as we increasing the number of fins the effectiveness is increasing at the steady rate, with maximum effectiveness $\epsilon = 3.63$ for the case of N=16 and then the effectiveness starts to decrease.

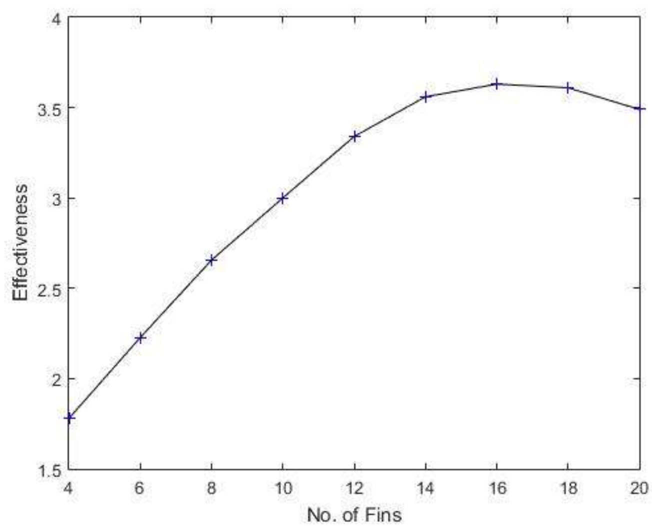


Fig. 19: Effectiveness vs No. of fins.

4. Conclusion

- From the numerical simulation of the various cases it can be concluded that the fluctuation of temperature is very gradual and slow as it should be in a laminar flow natural convection
- The analytical and numerical solution had an error of 0.14% only.
- The Rayleigh number decreases as the number of fins increases for the case of laminar flow natural convection.
- On comparing the Nusselt number values according to the Churchill's equation and from the CFD simulations, the error was only 1.74%.
- The plots of heat flux showed that optimization for the given cases is very limited.

- The overall heat transfer is increasing with number of fins up till 16 fins and then it decreases. Thus, the total heat transfer can be optimized for such natural convection cases and the maximum heat transfer is for 16 fins at this given length.
- Lastly, the variation of effectiveness concludes that the effectiveness can also be optimized for practical use and the maximum value of effectiveness is achieved in the case of 16 fins.
- The research showed us the temperature contours, heat flux, convective heat transfer coefficient and effectiveness of hollow cylinder with annular fins with constant temperature conditions and it can be stated that the values change gradually. Moreover, the total heat transfer and effectiveness can be maximized and one should keep these data in mind before designing similar thermal systems in industrial applications.

References

[1] J. R. Welling, C. B. Wooldridge, "Free convection heat transfer coefficients from rectangular vertical fins," *J Heat Transf.*, vol. 87, pp. 439-44, 1965.

[2] F. Harahap, F. McManus, "Natural convection heat transfer from horizontal rectangular fin arrays," *J Heat Transf.*, vol. 89, pp. 32-38, 1967.

[3] B. Singh, S.K. Dash, "Natural convection heat transfer from a finned sphere," *Int J Heat Mass Transf.*, vol. 81, pp. 305-24, 2015.

[4] A. D. Kraus, A. Aziz, J. Welty, *Extended surface heat transfer*. New York: John Wiley & Sons, 2001.

[5] J. R. Senapati, S. K. Dash, S. Roy, "Numerical investigation of natural convection heat transfer over annular finned horizontal cylinder," *Int J Heat Mass Transf.*, vol. 111, pp. 149-59, 2016.

[6] J. R. Senapati, S. K. Dash, S. Roy, "3D numerical study of the effect of eccentricity on heat transfer characteristics over horizontal cylinder fitted with annular fins," *Int J Heat Mass Transf.*, vol. 108, pp. 28-39, 2016.

[7] S.A. Nada, M.A. Said, "Effects of fin geometries, arrangements, dimensions and numbers on natural convection heat transfer characteristics in finned-horizontal annulus," *Int J Heat Mass Transf.*, vol. 137, pp. 121-37, 2019.

[8] S. Acharya, S.K. Dash, "Natural Convection heat transfer from a horizontal hollow cylinder with internal longitudinal fins," *Int J Heat Mass Transf*, vol. 134, pp. 40-53, 2018.

[9] D. Sun, Q. Shen, Y. Xu, T. Jin, X. Zhao, N. Zhang, K. Wu, Z. Huang," Natural Convection heat transfer along vertical cylinder heat sinks with longitudinal fins," *Int J Heat Mass Transf.*, vol. 100, pp 457-64, 2015.

[10] S. W. Churchill, H. H. S. Chu, "Correlating equations for laminar and turbulent free convection from a horizontal cylinder," *Int J Heat Mass Transf.*, vol. 18, pp. 1049-53, 1975.