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Implementing a Preconditioning Technique in RANS Equations to Accelerate the Code Convergence Rate

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Abstract - In this paper are presented a preconditioning technique to be implemented in a three-dimensional explicit compressible code to solve a turbulence flow to steady state regime. A local preconditioning technique with accurate predictions of mixed speed regimes is implemented in the original code, however, for low flow Mach numbers in the boundary layer region the numerical accuracy is lost to the preconditioning code. To improve the numerical solution are suggested a new limit to the preconditioning sensor based on a pressure sensor and is established an explicit flux function to evaluate the preconditioning sensor in the cell fluxes. The preconditioning code is validated for a supersonic case in nozzle and then to a subsonic case is studied the convergence rate for a low Mach number flow. Numerical solutions demonstrated that the changes applied in the original code improves the accuracy and robustness of the code for low speed flows.

***Keywords*:** CFD, Preconditioning, Eigenvalues, Mach number, All speed.

Nomenclature

Q	Vector of conservative variables.
E_e, F_e, G_e	Flux vectors of inviscid part.
E_v, F_v, G_v	Flux vectors of viscous part.
V	Volume.
ν	Kinematic viscosity coefficient.
$\hat{\nu}$	Modified eddy viscosity coefficient.
S	Source vector.
u, v, w	Velocity in Cartesian coordinates.
ρ	Density.
p	Pressure.
T	Temperature.
a	Sound speed.
E	Total stagnation energy.
e	Internal energy.
γ	Ratio of specific heats.
τ_{ij}	Shear-stress tensor.
α	Free parameter constant.
β	Preconditioning sensor.
M	Mach number.
Re	Reynolds number.
δ_{ij}	Kronecker delta.
c_{b1}, c_{b2}, c_{w1}	Constants of the Spalart-Allmaras model.
f_{t2}, f_w, σ	Constants of the Spalart-Allmaras model.
d	Distance to the closest surface.
Pr	Prandtl number.

1. Introduction

In the last years, with the computational improvements are increased the application of the computational tools to simulate stresses, fluid flow, heat transfer, and others, replacing some experiments which decreases the costs in the project design. Computational Fluid Dynamics (CFD) developments in the last two decades turned CFD as a powerful tool to simulate complex fluid flows [1].

The solution of compressible equations for low Mach number flows has a stiffness in the convergence rate, due to the large disparity between the acoustic waves and the wave speeds. Chorin [2] to improve that developed the artificial dissipation method adding a pressure derivative in the continuity equation and a multiplicative parameter (β). The resultant system of equations became a symmetric hyperbolic system, accelerating the convergence rate, however, the time accuracy is loss.

Choi and Merkle [3] and Turkel [4] following the work of Chorin [2] introduced the preconditioning technique, applying the pressure time derivative in the momentum equations and a new multiplicative parameter (α), this system of equations may be symmetrized by a change of variables. As a result, the preconditioning system of equations is hyperbolic in time and well-conditioned in the incompressible limit, ensuring the solution convergence. As the solution condition number (ratio between the highest and lower eigenvalues) influence directly in the convergence rate, several authors developed preconditioning techniques to decrease the solution condition numbers for low-speed flows [4].

Two types of preconditioning techniques are studied by several authors [5-7], the global and local preconditioning. The main difference between global and local preconditioners is how the preconditioning sensor is evaluated, in the global preconditioning the sensor is based on the global velocity scale and in the local is based on the local velocity scale. Although the global preconditioners have shown an increase in the code robustness, for complex flows is not enhanced the accuracy of the solution which does not occur in the local preconditioners.

Turkel [8] and Allmaras [9] investigating local preconditioners proves that, for a flow Mach number lower than 0.1 the preconditioning technique based on the primitive variables had lower instabilities, increasing the convergence rate. Hence, the authors suggested changes in the dependent variables to the conservative to the primitive then, the resultant matrix is obtained multiplying the preconditioning matrix for the Jacobian matrix improving the code robustness.

Local preconditioners have shown a robustness loss near stagnation points, investigations made in [10] demonstrates that when the preconditioning sensor goes to zero the solution eigenvectors became non-orthogonal inducing the transient amplification growth. Note that an efficient preconditioning technique should guarantee when the local speed goes to zero the value of the preconditioning sensor does not go to zero. Looking for a robust preconditioning technique Turkel, Vatsa and Radespiel [11] established a simple cut-off to the preconditioning sensor as a function of the free stream Mach number. With that, the authors obtained an efficient preconditioner to compute mixed speed regimes which calculates the lift and drag forces in a three-dimensional wing with accuracy.

A new study developed by Darmofal and Siu [12] to avoid the transient amplifications near stagnation points proposed a preconditioner limit based in the pressure sensor and a local flux preconditioner to be implemented in a block Jacobi algorithm. The limit imposed and the block Jacobi algorithm showed increases in the code convergence rate, as well as improves the robustness for low Mach number flows.

In the present paper are shown the implementation of the preconditioning method of Turkel, Vatsa and Radaspiel [11] applying a new limit to the preconditioning sensor in an explicit compressible code originally developed by Tomita [13] in the Reynolds-averaged Navier-Stokes equations in the Spalart-Allmaras turbulence model [14]. The original and preconditioning codes are validated to a supersonic flow with the experimental data obtaining accurate solutions [15]. To test the behaviour of the preconditioning code the pressure inlet is decreased to decrease the Mach number flow. For the subsonic case the preconditioning code improves the code accuracy and accelerates the convergence rate.

2. Mathematical Formulation

The Reynolds-averaged Navier-Stokes equations including the Spalart-Allmaras turbulence modelling in the Cartesian form is,

$$\iiint_V \frac{\partial Q}{\partial t} dV = -\iint_S \left[\frac{\partial(\overrightarrow{E_e} - \overrightarrow{E_v})}{\partial x} + \frac{\partial(\overrightarrow{F_e} - \overrightarrow{F_v})}{\partial y} + \frac{\partial(\overrightarrow{G_e} - \overrightarrow{G_v})}{\partial z} \right] dS - \iiint_V ST dV, \tag{1}$$

where, $\vec{Q}$ is the vector of conservative variables, $\vec{E}$, $\vec{F}$, and $\vec{G}$ are the flux vectors, the subscript e indicates the inviscid part (Euler) and the subscript v indicates the viscous part,

$$\vec{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \\ \rho \hat{v} \end{bmatrix}, \overrightarrow{E_e} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \\ (\rho E + p)u \\ \rho u \hat{v} \end{bmatrix}, \overrightarrow{F_e} = \begin{bmatrix} \rho v \\ \rho v u \\ \rho v^2 + p \\ \rho v w \\ (\rho E + p)v \\ \rho v \hat{v} \end{bmatrix}, \overrightarrow{G_e} = \begin{bmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w^2 + p \\ (\rho E + p)w \\ \rho w \hat{v} \end{bmatrix},$$

$$\overrightarrow{E_v} = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{xx} u + \tau_{xy} v + \tau_{xz} w + \gamma \left(\frac{\mu}{Pr}\right) \frac{\partial e_x}{\partial x} \\ (\nu + \hat{v}) \frac{\partial \hat{v}}{\partial x} \end{bmatrix},$$

$$\overrightarrow{F_v} = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ \tau_{xy} u + \tau_{yy} v + \tau_{yz} w + \gamma \left(\frac{\mu}{Pr}\right) \frac{\partial e_y}{\partial x} \\ (\nu + \hat{v}) \frac{\partial \hat{v}}{\partial y} \end{bmatrix},$$

$$\overrightarrow{G_v} = \frac{1}{Re} \begin{bmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ \tau_{xz} u + \tau_{yz} v + \tau_{zz} w + \gamma \left(\frac{\mu}{Pr}\right) \frac{\partial e_z}{\partial x} \\ (\nu + \hat{v}) \frac{\partial \hat{v}}{\partial z} \end{bmatrix}.$$

The source vector is,

$$ST = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -p \\ S \end{bmatrix}.$$

The S is defined as,

$$S = c_{b1}(1 - f_{t2}) S_{ij} \hat{v} - \left[c_{w1} f_w - \frac{c_{b1}}{\kappa^2} f_{t2}\right] \left(\frac{\hat{v}}{d}\right)^2 + \frac{1}{\sigma} c_{b2} \left(\frac{\partial \hat{v}}{\partial x}\frac{\partial \hat{v}}{\partial x} + \frac{\partial \hat{v}}{\partial y}\frac{\partial \hat{v}}{\partial y} + \frac{\partial \hat{v}}{\partial z}\frac{\partial \hat{v}}{\partial z}\right). \tag{2}$$

The constants of the turbulence model are specified in [14]. All variables are dimensionless, ρ is the flow density, u, v, and w are the velocity in Cartesian coordinates, $\hat{v}$ is the turbulence viscosity, E is the total stagnation energy defined by Eq. (3),

$$\mathrm{E} = \mathrm{e} + \frac{1}{2}(u^2 + v^2 + w^2). \tag{3}$$

The static pressure, *p*, is,

$$\mathrm{p} = (\gamma - 1)\left[\mathrm{E} - \frac{1}{2}(u^2 + v^2 + w^2)\right], \tag{4}$$

μ is the laminar viscosity, *Pr* is the Prandtl number, *Re* is the Reynolds number, and τ is the viscous stress tensor,

$$\tau_{ij} = 2\mu\left(S_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right), \tag{5}$$

where,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \tag{6}$$

Writing the Right-hand-side of Eq. (1) as $RHS(Q)$,

$$\frac{\partial \vec{Q}}{\partial t} = -RHS(Q), \tag{7}$$

Turkel, Vatsa and Radaspiel [11] suggested a preconditioning technique where the resultant preconditioning matrix is the result of the Jacobian matrix which modifies the conservative to the primitive variables and the preconditioning matrix. Multiplying the vector of conservative variables (Q) by the preconditioning matrix (Γ^{-1}) and the Jacobian matrix,

$$\Gamma^{-1}\frac{\partial Q}{\partial q}\frac{\partial q}{\partial t} = -\mathrm{RHS(Q)}. \tag{7}$$

Eq. 7 could be written as,

$$\frac{\partial \vec{q}}{\partial t} = -\Gamma\frac{\partial q}{\partial \mathrm{Q}}[RHS(Q)], \tag{8}$$

Γ is the preconditioning matrix is written,

$$\Gamma = \begin{pmatrix} \beta^2 & 0 & 0 & 0 & 0 \\ -\dfrac{(\alpha+1)u}{\rho} & 1/\rho & 0 & 0 & 0 \\ -\dfrac{(\alpha+1)v}{\rho} & 0 & 1/\rho & 0 & 0 \\ -\dfrac{(\alpha+1)w}{\rho} & 0 & 0 & 1/\rho & 0 \\ \hat{T}\left[\beta^2 + \dfrac{W^2}{2} - \dfrac{a^2}{\gamma-1}\right] & \hat{T}u & \hat{T}v & \hat{T}w & \hat{T} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

α is a free parameter set in a range between 0 and 1, a is the sound speed and, $\hat{T}$ is dimensionless temperature defined by the Eq. (9).

$$\hat{T} = \frac{(\gamma - 1)T}{\gamma p}, \tag{9}$$

W^2 is calculated by the Eq. (10),

$$W^2 = (u^2 + v^2 + w^2), \tag{10}$$

β is the preconditioning sensor established by Turkel, Vatsa and Radaspiel [11] as,

$$\beta^2 = \min\left\{max\left[K_1(u^2+v^2+w^2)\left(1+\frac{1-M_0^2}{M_0^4}\right), K_2^2(u^2+v^2+w^2)\right], a^2\right\}, \tag{11}$$

M_0 is the cut off Mach number, K_1 is a constant set between 1.0 and 1.1 and, K_2 is a constant set between 0.4 and 1.0. Darmofal and Siu [12] did a Fourier analysis of the transient amplifications near stagnation points, then the authors suggested a new limit to the preconditioning sensor based in the local pressure. Eq. (12) show the new limit,

$$\beta_{lim_{new}} = \frac{|p_R - p_L|}{\rho a^2}. \tag{12}$$

Where R and L corresponds to the right and left sides of the control volume. For each cell of the control volume we have,

$$\beta_{cell}^2 = \max_{k=1}^{k=6}(\beta_{face}^2)_k, \tag{13}$$

β_{face} is,

$$\beta_{face}^2 = \min\left[1, \max\left(\beta_L^2, \beta_R^2, \beta_{lim_{new}}^2\right)\right]. \tag{14}$$

Also, Darmofal and Siu [12] proposed to implement a flux preconditioner, the preconditioning sensor flux (β_{flux}) for each volume cell is calculated with the Eq. (15),

$$\beta_{flux}^2 = \max\left(\beta_{cell_R}^2, \beta_{cell_L}^2\right). \tag{15}$$

The value of α is redefined as a function of the cut off Mach number,

$$\alpha = \frac{(1 - M_0^2)}{M_0^2}. \tag{16}$$

Turkel, Vatsa, and Radaspiel [11] suggested the Eq. (17) to calculate the time step in the one-dimensional case to the preconditioning cells.

$$\Delta t = \frac{CFL}{\frac{1}{2}\left[|W|(1-\beta^2) + \sqrt{u^2(1-\beta^2) + 4\beta^2 a^2}\right]}\Delta x. \tag{17}$$

3. Numerical Simulations

Tomita [13] developed a three-dimensional compressible code which solves Euler and Navier-Stokes equations. The finite volume method with a centered scheme is applied to do the spatial discretization and to guarantee the numerical stability is added an artificial dissipation method [16]. The five-steps Runge-Kutta scheme is used to make the time integration [17], also the Implicit Residual Smoothing technique of Jameson, Schmid and Turkel[16] is applied in the code to accelerate the convergence rate.

To implement the preconditioning technique in the current code, it is necessary the total residual (RHS) composed by the inviscid, viscous, source, artificial dissipation and implicit residual smoothing parts [18]. Then, the resultant matrix is multiplied by the preconditioning matrix defined above. The time-step is done with the Runge-Kutta scheme through the primitive variables. After that, the vector of primitive variables is transformed into the conservative variables.

The turbulent flow in a nozzle is simulated for supersonic and subsonic flows. The preconditioning code is validated for the supersonic flow with the experimental data based in [15]. To test the preconditioning technique performance is applied a low-pressure ratio submitting the nozzle to a low speed flow, unfortunately it is not encountered experimental data to validate this case.

3.1. Nozzle

A nozzle is submitted to turbulent flow and was numerically simulated to the original and preconditioning code. The boundary conditions applied were: pressure inlet, pressure outlet and symmetry. The original and preconditioning code is validated for a supersonic case with experimental data [15] applying the static pressure ratio of 5. The stagnation pressure in the outlet is 101,325 Pa and the stagnation temperature in the outlet is 300 K. The CFL adopted is 0.12 and the ratio between the flow turbulent viscosity and the modified viscosity was 10. An unstructured mesh with 81,200 nodes, 63,180 volumes and 35,142 2D elements is used. A sketch of the mesh is shown in Fig. 1.

Fig. 1: A mesh sketch used to simulate a laminar flat plate.

The supersonic flow in a nozzle was validated with the pressure distribution in function of the nozzle length (x) [15]. Figs. 2 and 3 present the pressure distribution and the Mach contours to the original and preconditioning code.

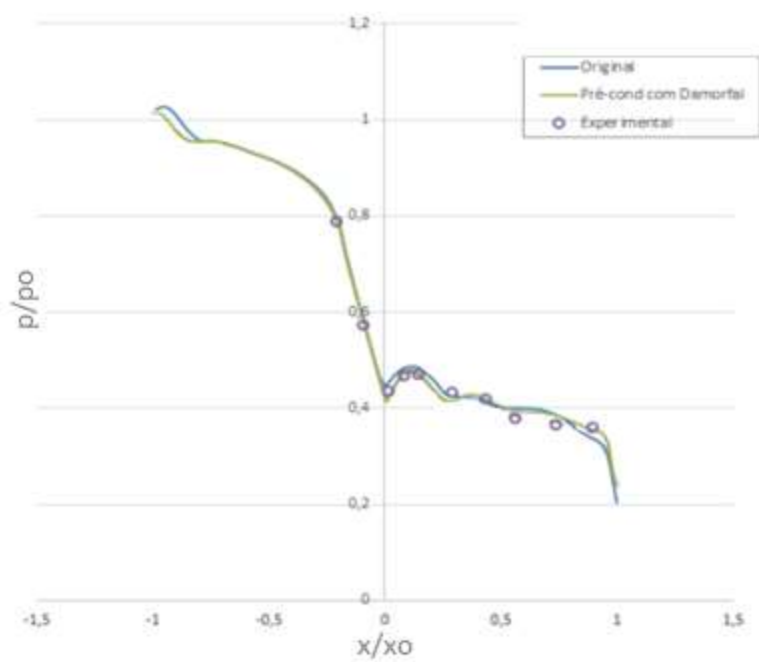
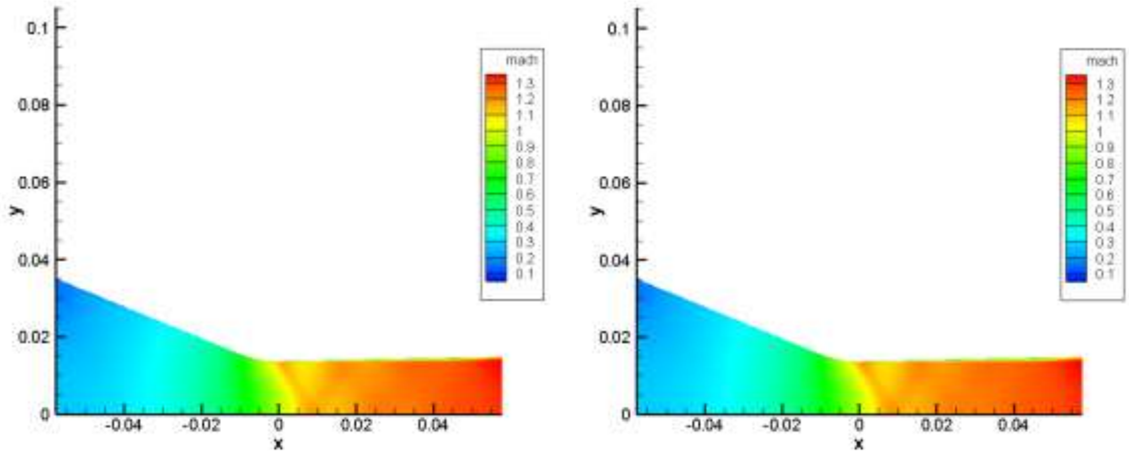


Fig. 2: Experimental and numerical pressure ratio distribution to the original and preconditioning code.



(a)Original. (b)Preconditioning.

Fig. 3: Mach contours to the original and preconditioning code.

The pressure distribution present in Fig. 2 shows close results with the experimental data for the original and preconditioning code demonstrating the numerical solution accuracy. The Mach contours (Fig. 3) had an approach behaviour for the original and preconditioning code with the formation of the oblique shock waves at the divergent nozzle section. Fig. 4 shows the logarithm residual histories.

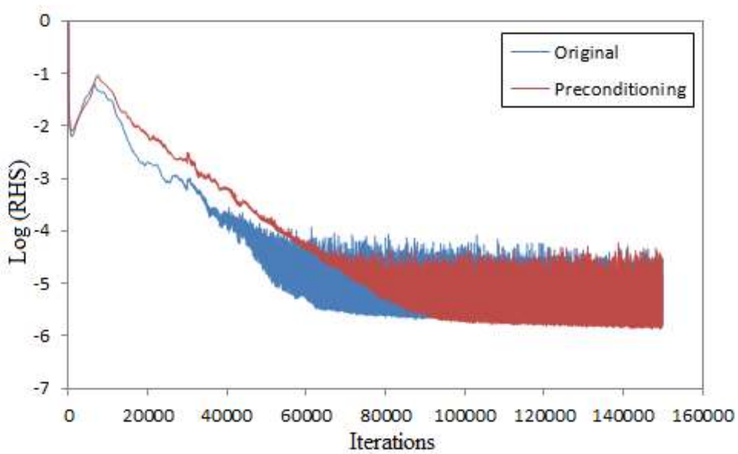
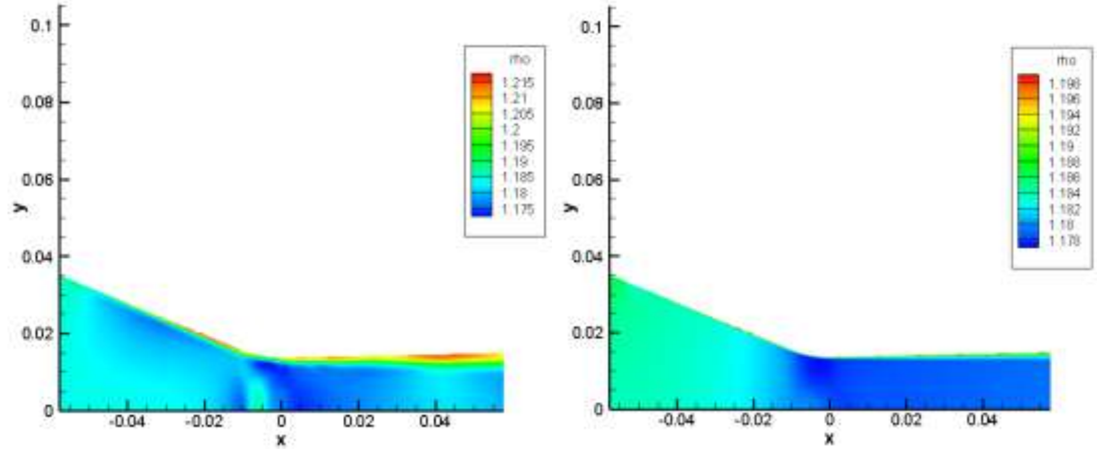


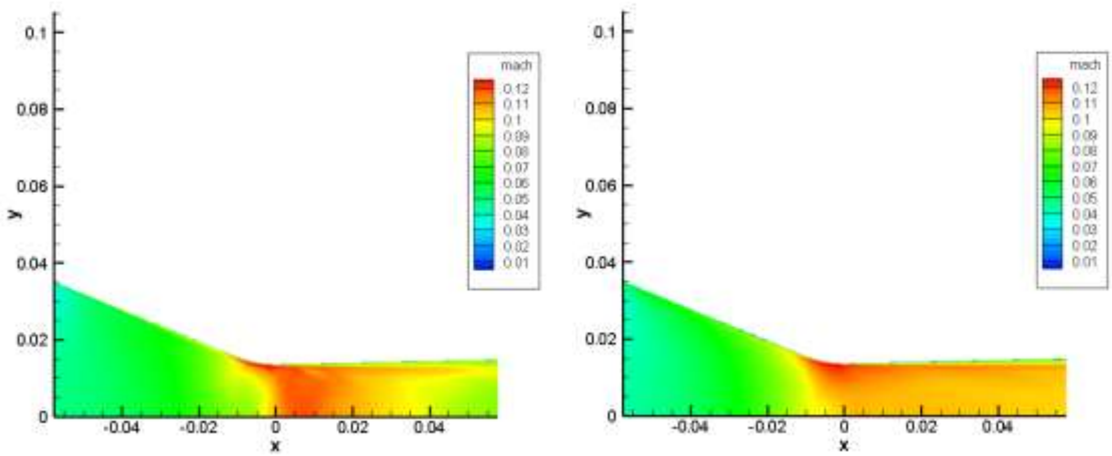
Fig. 4: Log residual histories to a supersonic nozzle.

Looking to test the original and preconditioning code for low-speed flow, the inlet static pressure is decreased to 102,036.4 Pa. Outlet static pressure, static temperature, turbulent intensity and CFL are the same described above. Figs. 5 and 6 show the density and Mach contours for the original and preconditioning code.



(a)Original. (b)Preconditioning.

Fig. 5: Density contours to the original and preconditioning code.



(a)Original. (b)Preconditioning.

Fig. 6: Mach contours to the original and preconditioning code.

Analysing the density contours in the Fig. 5 the original code showed a variation of the density values of 4% mainly in the boundary layer region, also is present spots in the numerical solution indicating a deterioration of the numerical solution for the low speed flow. The density contours of the preconditioning code had a variation of 2% as required when the flow Mach number is lower than 0.1. The Mach contours show in Fig. 6 had the same values of high Mach numbers to the original and preconditioning code, however for the original code present a solution without a uniform distribution of velocity in the divergent section demonstrating issues in the numerical solution for low Mach number flow. Fig. 7 shows the logarithm residual histories.

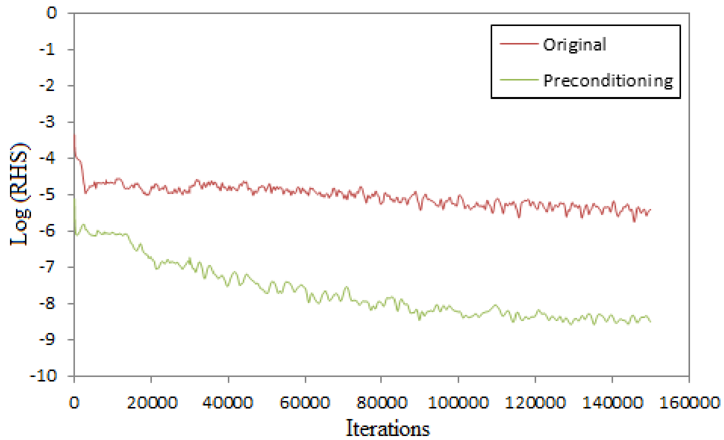


Fig. 7: Log residual histories to a nozzle for subsonic flow.

The residual logarithm histories show a faster convergence rate to the preconditioning code due to the large disparity between the flow speed and the sound speed which causes a stiffness in the convergence [12]. The preconditioning code converges with 4400 iterations and the original code converges with 6050 iterations.

4. Conclusion

In the present work, the preconditioning technique is implemented in a 3D Reynolds-averaged Navier-Stokes equations with the Spalart-Allmaras turbulence model. After research in the literature, the local preconditioning method of Turkel, Vatsa, and Radaspiel [11] had applied due to the accuracy obtained for mixed flows to predict lift and drag coefficients on airfoils. Looking to avoid low values of preconditioning sensor being implemented, a new limit with is a function of the flow pressure and an explicit flux function suggested by [12].

The turbulent flow is simulated in a nozzle for supersonic and subsonic flows. The preconditioning code is validated with the experimental data for a pressure ratio of 5 [15] proving the code accuracy predicting compressible code. To study the behaviour of the original and preconditioning code has decreased the inlet static pressure obtaining a subsonic flow. The numerical solution obtained to the original code shows a degrading in boundary layer regions where the flow Mach number goes to zero. The preconditioning code improves the numerical solution and increases the code convergence rate for low speed flow.

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